What is shaft stiffness?

Overhung centrifugal pumps utilise a shaft to connect the motor to the impeller. This shaft is held in place by bearings. The shaft passes through the pump housing (volute) and has the impeller pumping the fluid attached to the end of it. There is a seal between the shaft and the volute which prevents any of the pumped liquid from escaping.

![Diagram of centrifugal pump showing axial and radial loads, deflection and wear on bearings.]

Figure 1 – Centrifugal pump showing axial and radial loads, deflection and wear on bearings

When the pump impeller is rotated there is an axial load exerted by the fluid and radial loads exerted from the motor. These loads will cause the shaft to flex and bend. The stiffer the shaft the less bending occurs. In general a larger diameter shaft will bend less than a smaller diameter shaft. Also the material composition of the shaft will play a role in how much it will bend for a given load. Shaft stiffness is the measurement of how much a given diameter of shaft bends under load.
Why is shaft stiffness important?

The loads exerted by the motor and impeller on the shaft cause small radial deflections. If these deflections are too high then:

a) Wear occurs at the shaft seals making them prone to leakage, which for some chemicals can be very hazardous and/or requires expensive seal maintenance/replacement to be undertaken regularly.

b) Loads through the bearings are increased causing premature wear and if the loads through the bearing are higher than anticipated this may cause failure and overheating which in some environments could induce the risk of explosion.

Although a larger diameter shaft will generally be stiffer and deflect less for the same loads, pump manufacturers have to balance the diameter of the shaft with other design considerations:

a) Generally a larger diameter shaft requires larger and therefore more expensive seals.

b) The area available for the impeller blades at the impeller eye (where it is keyed to the shaft) may limit flow and reduce the efficiency.

c) Larger bearings increase the overall size and weight of the pump drive assembly incurring additional cost.

As such, the pump engineer’s challenge is to design a shaft that has the smallest possible diameter to save costs but one that will not flex beyond given tolerances and cause operating failures.

How do manufacturers calculate shaft stiffness?

API 610 11th edition Annex K gives a standardised method for calculating an overhung pump shaft flexibility index that can act as a baseline for comparing shaft flexibility between different pump designs.

For a shaft of two diameters $D_1$ under the seal sleeve and $D_2$ between the bearings (see Figure 2), the shaft stiffness is inversely proportional to what is generally termed the shaft flexibility index $I_{SF}$ – this is defined in the equation:

$$I_{SF} = \frac{L_1^3}{D_1^4} + \frac{L_1L_2^2}{D_2^4}$$

where $L_1$ is the overhang (centreline of impeller to line bearings) and $L_2$ is the bearing span.

Figure 2 – Stepped shaft showing relationship of shaft deflection and wear for different diameters
In rotor designs typical of refinery pumps, $D_2 > D_1$ and $L_2 > L_1$, the second term accounts for only about 20% of the total value of $I_{sf}$, so the convention widely adopted in the industry is to assess the overhung pump shaft stiffness using a shortened expression:

$$I_{sf} = \frac{L_1^3}{D_1^4}$$

Historically the industry would compare the value of $L_1^3/D_1^4$ to a ratio, typically 1.2x the lowest value of the pumps offered for service. If the pump vendor’s value of $L_1^3/D_1^4$ was within this range then the shaft was considered suitable for the application. If it fell outside this value it was considered cause for concern or further discussion.

However, because this assessment could only be applied to pumps of a similar size, API 610 11th edition recognises that other factors such as:

- Head
- Flow
- Speed

all have an impact on the suitability of a shaft stiffness value. For example a pump generating twice the head of another pump would require a stiffer shaft as would a pump operating at double the speed of another.

API 610 11th edition therefore has defined a “size factor”, otherwise known as $K_t$ and expressed by the equation:

$$K_t = \frac{Q \times H}{N}$$

Where $Q$ is the flow, $H$ is the head and $N$ is the rotating speed.

This size factor is calculated by taking into consideration the pumps BEP design at maximum impeller size and not simply the customer’s duty. This is done in case the pumps may be utilised for other duties at some point in the future.

Drawing on its many years of data from users, API 610 was able to plot the size factor for a broad selection of pumps running at different speeds that had proven reliable in service against their $L_1^3/D_1^4$ shaft stiffness value. Using a logarithmic scale, a line of best fit was found that could be used to evaluate a reliable shaft design for any pump. This line of best fit was converted into the formula:

$$I_{sf} = 32 \times K_t^{-0.76}$$

where

$$K_t = \frac{Q \times H}{N}$$

$(Q$ is the flow, $H$ is the head and $N$ is the rotating speed).

This formula now effectively defines a new benchmark value for an application against which customers can compare different manufacturers’ values of shaft stiffness (i.e. their $L_1^3/D_1^4$).

The value of $I_{sf}$ given by this formula can be readily calculated by the customer by reading off the maximum impeller BEP values from the manufacturer curves for a given pump. However, only the manufacturer can provide the value of $L_1^3/D_1^4$ for a specific shaft as the detailed specifications used in this calculation would generally only be known by them.

API 610 11th edition requires manufacturers (where requested) to provide their specific value of $L_1^3/D_1^4$ to enable comparison to the value calculated using the new API 610 11th edition $I_{sf}$ benchmark formula.
What do these numbers mean to a customer?

If a manufacturer’s value for L3/D4 entered on the API 610 datasheet is less than 120% of the value of I_{SF} calculated using the new formula there should be no cause for concern. Historical analysis has shown that L3/D4 values of less than 120% of the new I_{SF} benchmark value will not cause operational problems.

Where L3/D4 is greater than 120% of the new I_{SF} value, API 610 11th edition recommends further consideration. This does not mean that the design should be rejected outright but that further consideration should be given.

What should we consider when L3/D4 is more than 120% of the new I_{SF} benchmark value?

Some of the factors to consider when L3/D4 is greater than 120% of the new I_{SF} benchmark value include:

- How often is the pump running – frequently or infrequently?
- Is it a critical duty pump?
- Can a more forgiving seal be fitted which allows for greater shaft deflection in its design?
- How far above the 120% benchmark value is the manufacturers L3/D4?
- Is the shaft to be made from a special material as this may have a stronger inherent value?
- Is the customer’s operating point less than the pump’s duty point – for example is the operating point only 50% of the design head or design speed?
- Does the manufacturer have similar references without problems using the same design?

Any of these factors could mean that a shaft is suitable for the application although the shaft stiffness as measured by L3/D4 is greater than 120% of the new I_{SF} benchmark value.

In conclusion

API 610 11th edition is effectively aiming to provide a better way to evaluate different manufacturer’s shaft stiffness as measured by the L3/D4 value by taking into consideration the design head, flow and speed of the pump.

However, it is not as simple as saying that a particular L3/D4 value passes or fails. Generally a L3/D4 value less than 120% of the API 610 11th edition I_{SF} benchmark value is considered satisfactory but other factors do need to be considered outside of this value.

The process for evaluating designs since API 610 11th edition is for the manufacturers to provide the L3/D4 value for their design on the new API 610 11th edition datasheet and for the customer to compare this value to the new I_{SF} benchmark value they can calculate by using the formula:

\[
I_{SF} = 32 \times K_t^{-0.76} \text{ (SI Units)}
\]

or

\[
I_{SF} = 6200 \times K_t^{-0.76} \text{ (USC Units)}
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Currently API 610 11th edition does not demand manufacturers supply their L3/D4 values and so if there are concerns then the onus is placed on the customer to request the data and evaluate the results themselves.

By ensuring a shaft design meets the new I_{SF} benchmark value the benefits to the pump should be:

- Higher pump reliability
- Increased seal life
- Increased bearing life
- Increased shaft life
- Less vibration
- Less noise